22^{nd} *Computer Vision Winter Workshop*
Nicole M. Artner, Ines Janusch, Walter G. Kropatsch (eds.)
Retz, Austria, February 6–8, 2017

LBP Scale Space Origins for Shape Classification

Ines Janusch and Walter G. Kropatsch
Pattern Recognition and Image Processing Group
TU Wien, Austria
ines@prip.tuwien.ac.at, krw@prip.tuwien.ac.at

Abstract. *The LBP scale space serves as a shape representation, which thus allows not only for shape classification but also for an (approximate) reconstruction of the original shape. In this paper possible LBP scale space origins within a shape are evaluated. The influence of the LBP scale space center on the reconstruction quality is studied by computing the LBP scale space for every position inside a shape and by comparing the according reconstruction errors. For shape classification, the LBP scale space is further evaluated on the MPEG-7 dataset.*

1. Introduction

In this paper we present an evaluation of the locale binary pattern (LBP) scale space shape representation and experiments on shape classification. Biasotti et al. define shape descriptors to be able to identify a shape as a member of a certain class, while a shape representation also allows for an (approximate) reconstruction of the shape [2]. The discussed LBP scale space (LBPSS) serves as a shape descriptor. Augmented by polar coordinates however, the LBP scale space also serves as a shape representation, since in this case a reconstruction of the shape can be computed.

First experiments considering the LBP scale space were conducted by Janusch and Kropatsch in [9], before it was defined in more theory in [8]. The LBP scale space is based on LBPs which were originally proposed for texture analysis [15]. LBP codes however, capture not only local texture information, but also local topology. For texture analysis, description and classifications LBP codes are usually computed in a close neighbourhood around a pixel (most commonly pixels along a radius of 1, i.e. immediate neighbours of the center pixel). In order to use LBPs for shape description and shape classification, the local topology of larger neighbourhoods needs to be taken into consideration. For this purpose, a hierarchy of LBP codes - namely the LBP scale space - is computed over a range of radii and the persistence of LBP types is used as a shape representation.

Parameters for the LBPSS approach are the origin of the LBP scale space within a shape and the sampling of the range of radii. This paper studies the influence of the choice of the origin on the representational power of the LBP scale space and on its classification accuracy.

Yang et al. [20] give an overview of shape feature extraction techniques and distinguish information-preserving and non-information-preserving shape descriptors in this regard. Zhang and Lu [21] as well as Yang et al. [20] moreover distinguish contour-based and region-based methods for shape description and representation, and further subdivide them into structural and global techniques. Structural techniques employ shape primitives that represent the shape as segments while global methods represent the shape as a whole.

The evaluated LBP scale space combines the above mentioned categories: It examines the shape's contours as well as the shape's region. While it represents the shape globally, the shape may also be divided into intervals of topological persistence based on the LBP scale space, which can be considered shape segments.

Shape descriptors based on topological persistence are mostly obtained using a filtration of a space which gives a nested sequence of subspaces that begins with the empty and ends with the complete space [6]. Topological features appear (birth) or disappear again (death) for certain subspaces of the filtration. The interval between the birth and the death of such a feature, its lifetime, is called its persistence. Other shape descriptors based on topological persistence are for example size functions as described by Verri et al. [19] or persistence barcodes as presented by Carlsson et al. [4]. For shape retrieval Cerri et al. [5] presented a matching distance to compare shapes based on their persistence diagrams. Topological shape representations based on filtrations are in general dependent on the chosen filtration. Especially height functions (which are often used for filtrations) are not invariant to rotations of the shape. The LBP scale space in contrast is a rotational invariant shape representation.

The rest of the paper is structured as follows: Section 2 gives an introduction to the theory of LBP scale spaces, the influence of the LBPSS origin on the representational power is then discussed in Section 3. Section 4 gives on overview on shape classification in general and using the LBPSS representation. Experiments on shape classification based on the LBP scale space using the MPEG-7 dataset are presented in Section 5. Section 6 concludes the paper and gives an outlook to future work.

2. LBP Scale Space

LBPs were originally introduced for texture description and classification [15]. The LBP of one pixel is determined by comparing this pixel to a subsampled circular neighbourhood around it. For every such comparison the according position in a bit pattern is set to 1 if the grayvalue at the sampling point $g(x_i)$ is larger than or equal to the grayvalue of the center pixel $g(p)$ and to 0 otherwise (Fig. 1a and 1b):

$$s(x_i) = \begin{cases} 1 & \text{if } |g(p) - g(x_i)| \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Most LBP implementations and applications only consider the eight (see Fig. 1a) or four (above, below, left and right) direct neighbours to a center pixel. For the LBP computation two parameters may however be adjusted (see Figure 1c): P the number of sampling points the center pixel is compared with and R the radius of the circle around the center pixel along which the LBP is computed [16].

The bit pattern obtained as LBP not only captures texture but also encodes local topology: the neighbourhood around the center pixel may describe a (local) minimum or (local) maximum, a plateau, a slope or a saddle [7, 8]. These topological configurations of manifolds can be best illustrated on the example of topography: When considering the gray values in a grayscale image as elevations, a grayscale image forms a terrain model. In this case the topological configuration of minima is found at the bottom of valleys, maxima are located at mountain peaks - plateaus, saddles and slopes are in the same way intuitively defined. In an LBP these configurations of local topology can be distinguished by the number of bit changes in the LBP, this means by the number of transitions from 0 to 1 and vice versa in the bit pattern (e.g. a saddle is observed for LBPs with 4 or more such changes in the bit pattern) [8].

By varying the LBP radius and studying the evolution of the above defined topological classes over a range of such radii, the LBP scale space was introduced by Janusch and Kropatsch [9, 8]. The LBP scale space is defined in a continuous as well as in a discrete space. In the discrete case, the LBP scale space is obtained as follows: For a predefined center pixel (LBPSS origin) within a shape we compute the LBP for a range of radii. To fully cover a discrete shape a radius of 1 may be chosen as starting LBP radius and it may be increased by 1 until the shape is fully inscribed in this circle. The sequence of local topology configurations captured by the LBPs for each of the increasing radii gives a shape description. Figure 2a shows a small example shape and an according discrete LBP scale space. By augmenting the LBP scale space by polar coordinates, a shape representation is defined and an (approximate) reconstruction of the shape is enabled.

The continous LBP scale space with center p is defined as:

$$LBPSS(r, \phi) = \begin{cases} 1 & \text{if } |g(p) - g(r, \phi)| \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$LBPSS : [0, \infty) \times [0, 2\pi) \to \{0, 1\} \tag{2}$$

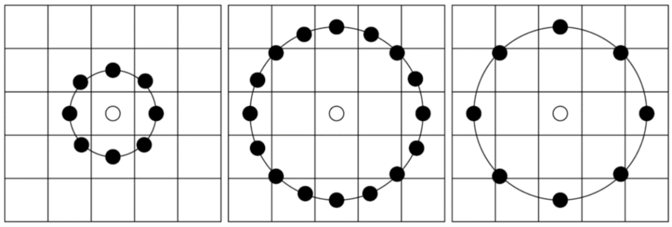
In the continuous case we observe osculation points along the shape's boundary for which the LBP circle's and the boundary's tangent in that point coincide. These osculation points are critical points since they describe a birth or death of a component, thus a change in topology. The interval of radii spanned in between two such consecutive critical points describes the lifetime (persistence) of a component. LBP circles of radii in such an interval all show the same topological class, since changes in topology only occur at critical points and topology

$x_1 \geq c$	$x_2 < c$	$x_3 < c$
$x_8 \geq c$	p	$x_4 < c$
$x_7 \geq c$	$x_6 \geq c$	$x_5 \geq c$

(a) comparison with neighbours

1	0	0
1	p	0
1	1	1

(b) bit pattern



(c) (P,r) = (8,1); (P,r) = (16,2); (P,r) = (8,2) - according to [16].

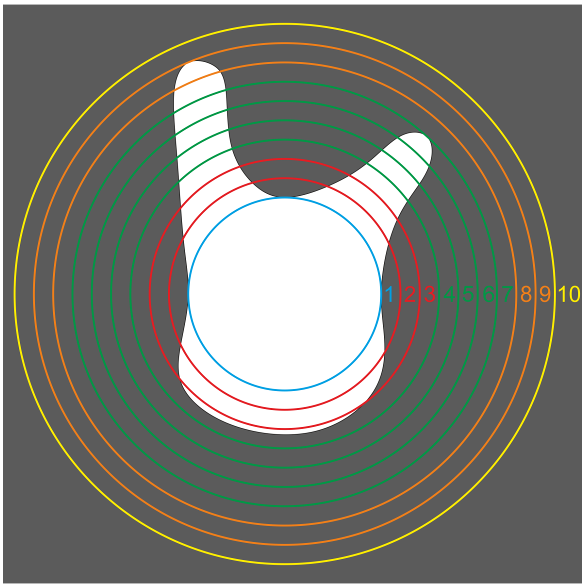
Figure 1: (a) and (b) LBP computation for center pixel p and (c) variations of the parameters P (sampling points) and r (radius).

persists in between two consecutive critical points. Fig. 2b shows an example for an example shape and an according continuous LBP scale space.

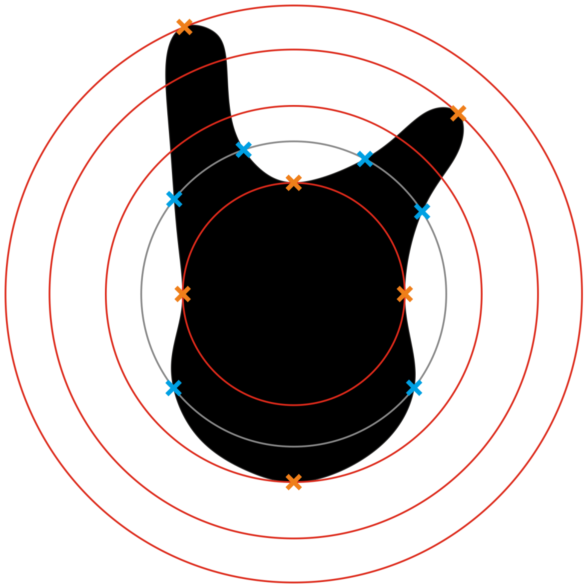
When implementing a discrete LBP scale space, it may start with a radius for which the circle is fully inscribed inside the shape. Therefore, the radius of this circle is at maximum the distance to the closest boundary, measured from the LBP origin. The LBP scale space is fully obtained once a radius is reached (according to the predefined sampling scheme) for which the shape is fully inscribed in the circle (i.e. at least half of the Euclidean diameter of the shape). For binary images (with a white foreground), the LBP scale space therefore always starts with a topological configuration of the type plateau - the circle is fully inscribed inside the shape and all pixels of the shape have the same colour. The LBP scale space terminates for those binary shapes with an LBP of type maximum since the origin of the LBP computation is white (foreground) and the circle is large enough to inscribe the whole shape, it is therefore located in the black background. In between these start and end states, the observed types of local topology may be of type plateau, slope or saddle.
The LBP scale space captures topological configurations which are given as transitions from 0 to 1 or vice versa in the LBP bit pattern. The topological configuration depends on the number of transitions only, the position of these transitions in the bit pattern is irrelevant. The LBP scale space thus is a rotation invariant shape representation. It is however not scale invariant, since for a predefined sampling scheme similarly formed but differently scaled shapes produced a different number of levels in the LBP scale space. Considering the LBP scale space as feature vector describing a shape, the feature vectors of these two similar but differently scaled shapes would be of different length. A normalization step applied to either the shapes or the sampling scheme is therefore needed as a preprocessing step in order to compare shapes of different scales.

3. Experiments: LBP Scale Space Origin

In previous experiments [8] the location of the maximum of the distance transformation of a shape and the location of the minimum of the eccentricity transformation of a shape were used as origins for the computation of the LBP scale space. In this experiment we evaluated whether these positions are suitable locations for LBP scale space origins and whether other positions within a shape should be considered as LBP scale space origins as well.
We used the Kimia 99 dataset [17] and computed the distance transformation (using the quasi-Euclidean metric) and the eccentricity transformation for all 99 shapes of this dataset for this experiment. Additionally, we computed all possible LBP scale spaces for every shape: every location within the shape served as origin, thus one LBP scale space was computed for every pixel of the shape. Every shape was then multpile times reconstructed - once for every LBPSS computed for this shape in the previous step. All reconstructions were compared to the original shape. For every pixel of the shape we stored the achieved reconstruction accuracy when reconstructing the shape based on the LBPSS with origin at this pixel.
Reconstruction errors arise in the discrete space whenever the inscribed LBP circle touches the shape boundary in a flat angle. In this case several osculation points (locate next to each other along the shape boundary) occur between the LBP circle and the shape boundary (respectively between their tan-



(a) discrete LBPSS



(b) continuous LBPSS

Figure 2: LBP scale space in the (a) discrete case for a regular sampling scheme. Blue (circle 1) = plateau, red (circle 2, 3) = degenerate saddle, green (circle 4-7) = non-degenerate saddle, orange (circle 8, 9) = slope, yellow (circle 10) = maximu. And the (b) continous case - the osculation points marked in orange along the red LBP circles are critical points at which the topology changes.

gents).
A low reconstruction error indicates a high quality shape representation, thus points within a shape for which the shape can be reconstructed well are suitable LBP scale space origins. By considering the reconstruction accuracy for all points within a shape, we obtain a map for each shape, that associates representational quality with each point of the shape. Figures 3i - 3l show such maps for four example shapes. We compared these maps and the obtained optimal origins of all shapes in order to obtain common rules for suitable LBP scale space origins. These optimal origins can so far only be determined through testing all possible locations within a shape, which is not a suitable way to determine LBP scale space origins for an actual shape representation. By comparing the reconstruction qualities of all possible origins of all shapes, we observed the following properties for suitable LBP scale space origins - they are located:

- inside the shape at some distance to the boundary but not at the boundary (cp. locations of maximum distance transformation)
- at central points of shapes, not in thin structures or convexities such as extremities for a humanoid shape (cp. locations of minimum eccentricity)

These observations agree with previous experiments in which various points inside a shape were tested as LBP origins and the location of maximum distance transform and minimum eccentricity transform provided the best reconstruction accuracy.

For applications in shape classification or matching, the LBP scale space origin should not be located at arbitrary positions for the individual shapes. To ensure good discrimination among the shapes, it should be determined according to defined rules since the LBP scale space representation also depends on the chosen origin.
Based on our above described observations the locations of maximum distance transformation, respectively minimum eccentricity transformation may serve as suitable origins. Figure 3 shows a comparison of distance transformation, eccentricity transformation and reconstruction accuracy using an LBP scale space for four example shapes. Out of these two transformations, the distance transformation however has a much lower computational complexity: The eccentricity transformation induces a constrained distance transformation for every pixel of the shape, i.e. for the eccentricity transformation the distance transformation is computed n times for a shape with an n pixel area. Thus, the complexity

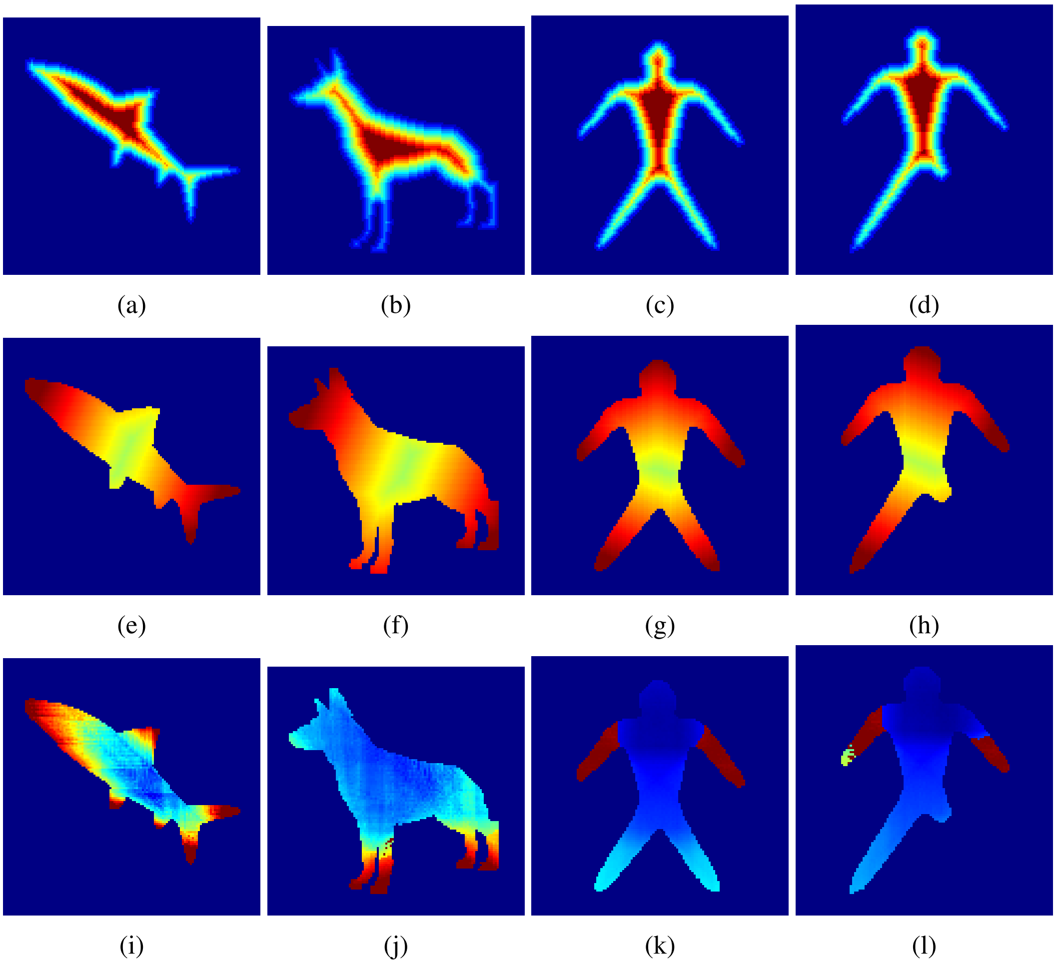


Figure 3: Comparison of distance transform (using the quasi-Euclidean metric) (a) - (d), eccentricity transform (e) - (h) and reconstruction error based on the LBP scale space (i) - (l). Blue indicates low, red high values.

may be up to $O(n^3)$ [10].
We therefore evaluated the location of maximum distance transformation for the shapes in the test dataset to see if this is a suitable location for an LBP scale space origin. The reconstruction error for an LBP scale space with origin at the location of maximum distance transform of each shape is at least 0.027% for the shapes in the Kimia 99 dataset. More than half of these shapes (54 out of 99) have less than 3% reconstruction error when reconstructed using an LBP scale space with origin at this location, 93% (92 out of 99) of these shapes have less than 10% reconstruction error. This shows that the location of maximum distance transform is indeed a suitable and easily obtainable origin for LBP scale space computations.

4. LBP Scale Space for Shape Classification

For 2D shape classification Hidden Markov Models (HMM) have for example been employed by Bicego et al. [3]. The authors use HMMs to compute similarities between an input shape and all other shapes in their dataset (so called representatives set). The actual classification task is then done in this obtained similarity space. Ling and Jacobs define the Inner Distance Shape Context (IDSC) as a shape descriptor that is robust to articulation [12]. The IDSC between to points on the boundary of the shape is defined as the shortest path inside the shape connecting the two points (cp. geodesic distance). Temlyakov et al. [18] propose two strategies to determine the similarity between two shapes: their first strategy focuses on shapes that can be decomposed into a compact base structure and strand structures (elongated shape parts attached to the base structure). The similarities are then determined individually for base

and strand structures. Their second strategy focuses on symmetrical shapes. The authors integrate both strategies into predefined shape classification methods, e.g. IDSC, to improve their performance.
The in this paper evaluated LBP scale space serves as a shape descriptor and a shape representation and can therefore be applied to shape matching and shape classification. Since LBP scale space representations for different shapes may vary in length depending on the shape itself, the scale of the shape and the origin of the LBP computation, the LBP scale space does not directly provide feature vectors that can be classified using approaches that assume a fixed number of features such as k-Nearest-Neighbours (kNN).
The string edit distance (Levenshtein distance) [11] proved to be a suitable metric for the comparison of LBP scale spaces. The Levenshtein distance measures the similarity of two strings as the minimum number of operations needed to transform one string into another one. These operations are insertion, deletion and substitution of a character. Within this paper the Levenshtein distance is used in its original definition - all operations are equally weighted and add a unit cost of 1 to the Levenshtein distance between two strings every time they are employed within a transformation of one string into another one.
By determining the Levenshtein distance between shapes based on their LBP scale spaces, a classification using kNN is enabled. Since a distance between shapes is given then, the k nearest shapes according to the edit distance can be considered for classification. Neuhaus and Bunke [14, 13] also employed the edit distance for strings as well as for graphs to enable classification and matching using kNN and support vector machines (SVM).

5. Experiments: Shape Classification

For this experiment the LBP scale space is evaluated on the MPEG-7 dataset [1]. This dataset consist of 1400 images in total which are grouped into 70 classes of shapes with 20 images per class[1]. For a classification using the LBP scale space the MPEG-7 dataset had to be slightly adapted since the LBP scale space is not suitable for a representation of shapes with holes in the foreground region. For a first experiment (*experiment no holes*) classes with holes in the shape were excluded. The classification was done on 51 instead of 70 classes of MPEG-7 in this experiment, the following 19 classes were excluded: bell, bird, butterfly, cattle, chicken, crown, deer, device6, dog, fly, frog, guitar, hat, horse, horseshoe, lizard, pocket, ray, turtle. For a second experiment (*experiment filled holes*) all 70 classes were included in the experiment but holes in the foreground regions of shapes were filled.
The discrete LBP scale space was then obtained at full pixel resolution, starting from a radius of 1 until a radius for which the whole shape is inscribed in the circle and sampling steps of 1 for this range of radii. The origin of the LBP scale space was located at the location of maximum distance transformation.
The LBP scale space of every shape was compared to the LBP scale spaces of all other shapes using the Levenshtein distance. The confusion matrix showing the distances between the shapes for the whole dataset (*experiment filled holes*) is given in Figure 4. For this matrix the origin is set at the upper left corner, smaller distance appear mostly for shapes within the same class, this is visible as dark blue patches along the diagonal. Each shape was then classified using leave one out cross validation by comparing to all other shapes. The class for every shape was determined using a kNN approach with $k = 1, 3, 5, 7$. Thus, as the class of the shape with the smallest Levenshtein distance, or respectively as the majority of classes of the 3, 5 or 7 closest shapes according to the Levenshtein distance, to the shape to be classified. The classification accuracy for these 4 options are given in Table 1.

Bicego et al. [3] evaluated their approach on a sub-set of the MPEG-7 dataset consisting of only 7 classes and 12 images per class. They report classification accuracies between 80.9% and 98.8% depending on the classification scheme (maximum-likelihood respectively kNN ($k = 1, 3$) based on HMMs were used). For the same subset of MPEG-7 the LBP scale space reaches a classification accuracy of 89% for both $k = 1$ and $k = 3$.
Ling and Jacobs [12] and Temlyakov et al. [18] evaluated their shape descriptors using bullseye testing which evaluates the application of shape descriptors to shape retrieval. Shape retrieval aims at selecting from a dataset all shapes, that are similar (to a certain degree) to an input shape. Bullseye testing is also taken as a measure for shape classification since a majority vote over the retrieved shapes may be used

[1] An overview of the dataset with an example image for each class can be found at `http://www.dabi.temple.edu/~shape/MPEG-7/dataset.html`

Table 1: Classification accuracy for the two experiments on the MPEG-7 dataset, using kNN with $k = 1, 3, 5, 7$.

	k=1	k=3	k=5	k=7
experiment no holes	82%	79%	75%	72%
experiment filled holes	79%	72%	68%	67%

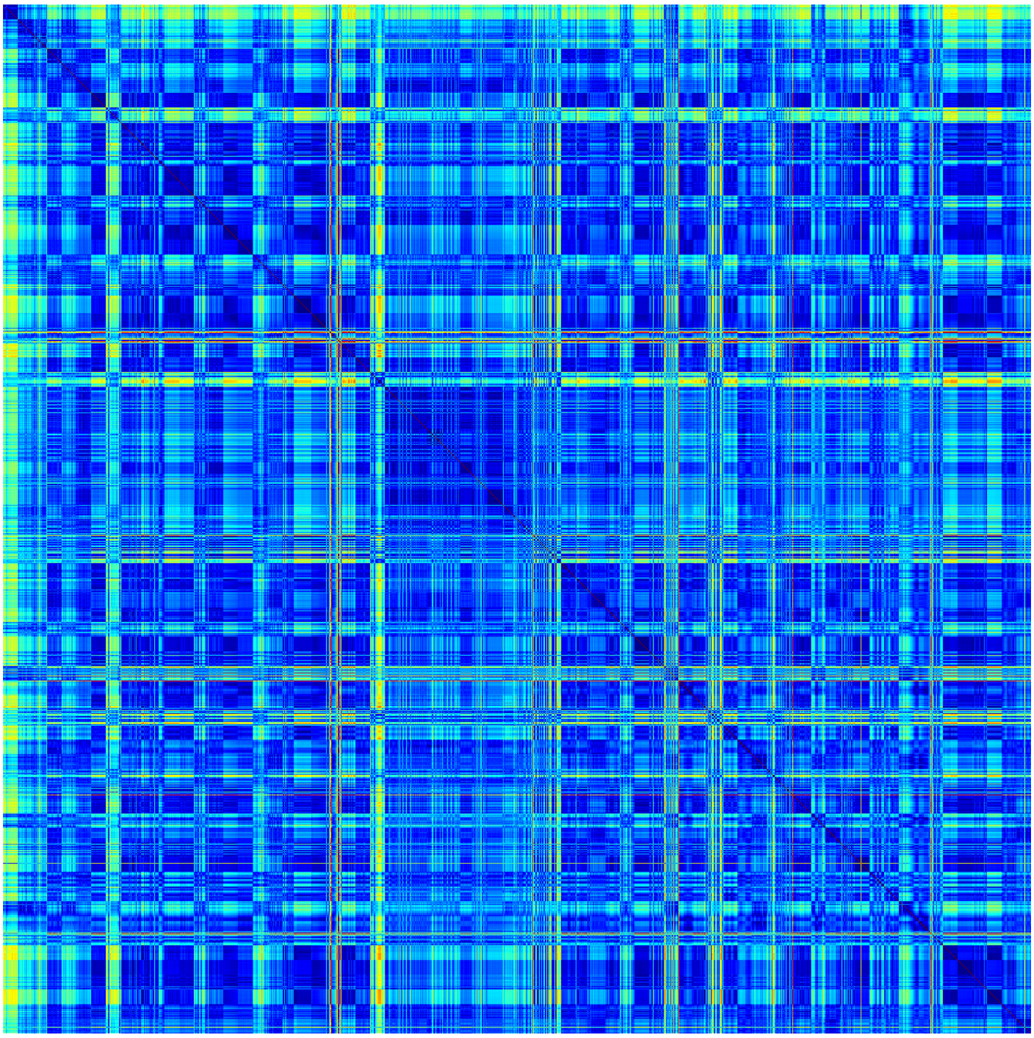


Figure 4: Confusion matrix showing the Levenshtein distances between the 1400 shapes of the MPEG-7 dataset (experiment filled holes). Blue indicates low distances, red high distances - the origin is located in the upper left corner.

to classify an input shape. For bullseye testing using the MPEG-7 dataset the following procedure is used:

- For every shape similarities/distances to all 1400 shapes are computed.
- The 40 closest (most similar) shapes are selected for each shape.
- Out of this 40 shapes the shapes that are in the same shape class as the input shape are counted.
- This count is summed for all shapes.
- The accumulated count over all shapes is set into relation to the maximum count (the maximum count per shape is 20, since there are 20 shapes in each class - thus 20×1400 in total). A maximum ratio of 100% may be reached.

Ling and Jacobs [12] and Temlyakov et al. [18] report high performances on the bullseye testing: with up 96% respectively 85% retrieval / classification rate.

Bullseye testing for the presented LBP scale space approach obtained lower retrieval / classification rates: for the MPEG-7 subset of shapes without holes in the foreground region (*experiment no holes*) 56% were reach, for all shapes with filled holes (*experiment filled holes*) 46%. Due to the to some extent high intra-class variability all 20 shapes of one class could only be obtained for 125 input shapes (*experiment filled holes*). The LBP scale space performs well for classifications applications for which a class is determined based on one or more most similar shapes. In this case even high variability within a class is handled well (see Table 1) since a small number of similar shapes can be detected reliably. For retrieval applications however this high variability in the dataset inhibits the retrieval of a large number of similar shapes. Planned extensions and improvements to the LBP scale space aim at solving this problem.

6. Conclusion and Future Work

The presented evaluation of the LBP scale space as a shape representation showed that the location of maximum distance transformation which can be easily computed for a binary shapes serves as a suitable origin for LBP scale space computations. Moreover, evaluations on the MPEG-7 dataset showed that the LBP scale space in combination with the Levenshtein distance can be used as shape descriptor for shape classification. For the application of shape retrieval improvements need to be made for the LBP scale space. The drawbacks that the LBP scale space currently entails for this application will be addressed in future work:

We plan to soften the parameters of the scale space by using closed concentric curves (that are able to adapt to the boundary of a shape) instead of the fixed concentric circles of the LBP computation. This more flexible shape representation is intended to be invariant to deformations (for example due to articulated movement) of a shape. It thus also allows for the retrieval of similar shapes showing higher deformations. Moreover, this representation allows to obtain a shape property - a polygon hull of a shape similar to a convex or concave hull of a shape.

Other future tests may include a fixed size LBP scale space. This means a fixed number of sampled radii within the interval of minimum and maximum LBP radius that are determined by the shape itself. Using such a fixed size LBP scale space creates a scale invariant shape descriptor based on the LBP scale space. However, a reconstruction of the input shape is only possible as approximation of this shape but converges to the input shape the smaller the sampling intervals are chosen (i.e. the larger the size of the LBP scale space is). The fixed size LBP scale space also allows to treat the LBP scale space as a feature vector of fixed dimensions and thus to classify without computing the Levenshtein distance but based on the LBP scale space directly using well known methods such as kNN or SVMs.

Acknowledgements

We thank the anonymous reviewers for their constructive comments.

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